JOURNAL OF THE OPTICAL SOCIETY OF AMERICA VOLUME 38, NUMBER 10 OCTOBER, 1948

The Measurement of the Velocity of Light by Signals Sent in One Direction

HERBERT E. IVES
Upper Montclair, New Jersey
(Received July 7, 1948)

The problem of measuring the velocity of light by signals sent in one direction, instead of the usual method, by signals sent out and back, is studied. This requires two clocks, and the necessary steps to set the two clocks are discussed. It is found desirable to distinguish between "velocity" in its elementary and traditional sense, and the "rod-clock-quotient" directly obtained by measurement with rods and clocks experiencing the Fitzgerald-Larmor-Lorentz contractions. The precise formula derived for the one-way measurement of the velocity of light involves two rod-clock-quotients. The use of one-way light signal measurements in the Lorentz transformations and the Special Theory of Relativity is discussed.

INTRODUCTION

SINCE the time of Galileo terrestrial measurements of the velocity of light have been made by out-and-back signals, whereby the time of transit is measured by a single "fixed" clock. This has been a tacit recognition of the difficulty of insuring the exact uniformity of rate and identity of setting of two separated clocks.

With the development of precision crystal clocks the measurement of the velocity of light by sending signals in one direction, timed by two separate clocks, could probably now be carried out with a fair degree of accuracy. While there appears to be no call to undertake such a measurement, the theory of the experiment presents some features of interest, with bearings on some fundamental questions in the defining and nomenclature of physical quantities, and hence deserves attention. In particular the problem emphasizes the importance of differentiating between various uses of the word "velocity" which have caused obscurity and confusion, notably in presentations of the Special Theory of Relativity.

GROUNDWORK

"Light" is here considered as a disturbance in a transmitting medium, traveling at a definite velocity "c" with respect to the medium, independent of the motion of the source with respect to the medium.[1] The term "velocity" is in this paper rigorously restricted to the ratio

$$\frac{\text{distance traveled in the medium}}{\text{time taken to travel the distance}},$$

where distance is measured by material rods *stationary in the medium*, and time is measured by a clock *stationary in the medium*.[2] This is equivalent to the alternative statement that distances and clock rates are measured by rods and clocks unaffected by their motion through the medium. This definition of velocity corresponds to its original meaning in physics before the Fitzgerald-Larmor-Lorentz contractions were postulated or experimentally established.[3] By it velocities are added arithmetically.

The Fitzgerald-Larmor-Lorentz contractions are taken in this paper to be functions of the velocity *with respect to the medium*, where velocity is defined in the above way.[4] Accordingly, a rod of stationary length l_0 becomes, when moving with velocity v (as above defined)

$$l = l_0[1-(v^2/c^2)]^{\frac{1}{2}},$$

and a clock of stationary frequency ν_0, assumes the frequency

$$\nu = \nu_0[1-(v^2/c^2)]^{\frac{1}{2}}.$$

MEASUREMENT OF THE VELOCITY OF LIGHT BY SIGNALS SENT OUT AND BACK

In order to emphasize the characteristics of one-way signal measurement, the theory of two-way or out-and-back measurement in terms of the above "Groundwork" should be reviewed.

In Fig. 1 let ab be the measuring platform: at a is a clock, at b a mirror. Let W be the velocity (in the above defined sense) of the medium past the platform. Let D' be the length of the platform, as measured by rods laid end to end on it. The true length, because of the F.L.L. contraction, is $D'[1-(W^2/c^2)]^{\frac{1}{2}}$. For t_1, the time of transit (by a clock stationary in the medium) of a light signal from a to b, we have

$$t_1 = \frac{D'[1-(W^2/c^2)]^{\frac{1}{2}}}{c-W}, \tag{1}$$

and similarly for the return signal,

$$t_2 = \frac{D'[1-(W^2/c^2)]^{\frac{1}{2}}}{c+W}, \tag{2}$$

[1] The frequent assertion that "the Michelson-Morley experiment abolished the ether" is a piece of faulty logic. When Maxwell predicted a positive result from the experiment he did so on the basis of *two* assumptions; the first, that the light waves were transmitted through a medium, the second, which was not realized until pointed out by Fitzgerald, that the measuring instruments would not be affected by motion. The nul result of the experiment proved *some* assumption made in predicting a positive result to be wrong. The experimental demonstration of the variation of measuring instruments with motion, in exactly the way to produce a nul result, shows that it was the second assumption alone that was wrong; leaving the evidence for a transmitting medium, as derived from aberrational and rotational phenomena, as strong, if not stronger, than ever.

[2] While more than one clock may be involved in a measurement, all clocks in a stationary medium can be given the same rate and setting by light signals, using for setting one-half the transit time out and back. Hence all clocks, being identical in indication, may be included under the above "a" clock. The question whether it is practically possible to determine that a clock or rod is stationary in the medium is no bar to using such rods and clocks in developing a theoretical argument.

[3] The demonstration of the contraction of clock rates is described in "An Experimental study of the rate of a moving atomic clock," H. E. Ives and G. H. Stilwell, J. Opt. Soc. Am. **28**, 215 (1938), and **31**, 369 (1941). For the proof of the contraction of lengths the Kennedy-Thorndyke experiment (Phys. Rev. **42**, 400 (1932)) serves. This assumed the contractions of length and showed that the nul result of the experiment demanded that clock rates should vary: With the just quoted positive establishment of the clock rate variation, the Kennedy-Thorndyke experiment proves the assumed length contraction.

[4] For the derivation of the F.L.L. contractions in terms of motion through the light transmitting medium from Maxwell's radiation pressure and the conservation laws, see "Derivation of the Lorentz transformations," H. E. Ives, Phil. Mag. **36**, 392 (1945).

so that

$$t_1+t_2=\frac{2cD'[1-(W^2/c^2)]}{c^2-W^2}. \qquad (3)$$

Now if t' is the total elapsed time as read by the clock at a, we have, from the F.L.L. contraction

$$t_1+t_2=\frac{t'}{[1-(W^2/c^2)]^{\frac{1}{2}}} \qquad (4)$$

from which we have finally

$$c=2(D'/t'). \qquad (5)$$

What we have obtained is that, although the velocity of light *relative to the platform* is $c-W$ out, and $c-W$ back, the quotient of rod reading to clock reading gives us the velocity of light *in the medium* where "velocity" has the meaning in the definition above. The F.L.L. contractions have eliminated W.

DISTINCTION BETWEEN VELOCITY AND ROD-CLOCK-QUOTIENT

The result just obtained, that measurements made with clocks experiencing the F.L.L. contractions do not give the velocity of light relative to the platform, brings out the necessity of differentiating between "velocity" as above defined, and the quantity obtained directly by the quotient of length and time measurement.

In order to be free of all ambiguity in further discussion, we shall, while restricting the term "velocity" to the definition above, designate by a new term the quotient of distance, measured by a rod stationary with respect to the platform, to the interval indicated by a clock (or clocks, as in the next section), to be used when rods and clocks are employed which experience the F.L.L. contractions. This we shall call the "*rod-clock-quotient*," and designate by the letter Q, using subscripts and change of type face to indicate the different clocks involved in any measuring operation.

The several Q's we shall need are as follows: Q_1 where a single clock, stationary on the platform, is used, Q_2 where two clocks, stationary on the platform, are used, q where a clock, moving with respect to the platform, is used.

In accordance with the above use of symbols, we have, from the previous section, for the

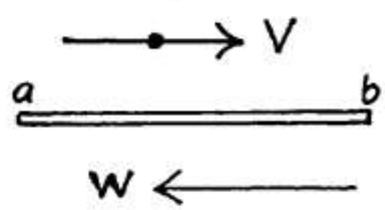

FIG. 1.

velocity of light, using out-and-back signals and a single fixed clock

$$Q_1=c. \qquad (6)$$

We shall find that this identification of c and Q_1, is not parallelled in the case of one-way signals and the corresponding Q_2.

MEASUREMENT OF THE VELOCITY OF LIGHT BY SIGNALS SENT IN ONE DIRECTION

In order to measure the velocity of light by a signal sent in one direction it is necessary to know its time of emission at one point (a in Fig. 1) and the time of its reception at another (b in Fig. 1). This means that we can no longer use one clock, we must have two, and the second must be set to run at the same rate as the first, and be set in some known relation to the first. Synchronism as to rate can be maintained, when both clocks are stationary on the platform, by periodic signals, assuming that distance of transmission does not alter the frequency of the signals. Similarity of setting cannot, however, be achieved by this means, without knowing the velocity of transmission with respect to the platform, which of course is the ostensible purpose of our measurement. Our only recourse is to set the second clock to agree with the first at the origin (a) and then move it to its stationary position at the far end of the platform (b), (or, what is equivalent, to use a third, moving, clock to carry the setting from one stationary clock to the other). The moved clock will have its rate changed because of its motion, and this will cause a change of setting. It is our next problem to investigate this change.

Consider the clock which is to take up its station, or to be used to set a clock, at b, as moved from a to b at a uniform rate, as measured by the number of divisions moved over, divided by the elapsed interval, *as measured by the clock* *itself*—there being no other available criterion for time evaluation.[8]

Designate by Y the velocity (in the sense of the above definition) of the clock. We then have, because of the F.L.L. contractions

$$Y=\frac{D}{\tau}=\frac{D'}{\tau'}\left[1-\frac{W^2}{c^2}\right]^{\frac{1}{2}}\left[1-\frac{(W+Y)^2}{c^2}\right]^{\frac{1}{2}}. \quad (7)$$

In this equation D' is the observed distance between reference points, by rods stationary on the platform; the distance D in the light transmitting medium is less by the factor $[1-(W^2/c^2)]^{\frac{1}{2}}$, where W is the velocity of the medium past the platform. τ' is the interval indicated by the moving clock; it is less than the time by a clock stationary in the medium by the factor $[1-(W+Y)^2/c^2]^{\frac{1}{2}}$.

Now according to the symbols above listed D'/t' is the rod-clock-quotient q for the moving clock. Putting this in (7) and solving for Y we get

$$Y=\frac{q[1-(W^2/c^2)]}{[1+(q^2/c^2)]^{\frac{1}{2}}+(qW/c^2)}, \quad (8)$$

and also

$$\tau'=\frac{\tau[1-(W^2/c^2)]^{\frac{1}{2}}-\dfrac{WD}{c^2[1-(W^2/c^2)]^{\frac{1}{2}}}}{[1+(q^2/c^2)]^{\frac{1}{2}}}, \quad (9)$$

which is the indication of the moved clock when it reaches b, and with which we set the clock fixed at b.

Since the clock left standing at a measures τ, the time taken by the moved clock to reach b, as $\tau[1-(W^2/c^2)]^{\frac{1}{2}}$, the difference in indication of the two clocks, Δ, is

$$\Delta=\tau[1-(W^2/c^2)]^{\frac{1}{2}}-\frac{\tau[1-(W^2/c^2)]^{\frac{1}{2}}-\dfrac{WD}{c^2[1-(W^2/c^2)]^{\frac{1}{2}}}}{\left(1+\dfrac{q^2}{c^2}\right)^{\frac{1}{2}}}$$

$$=\frac{\tau\left(1-\dfrac{W^2}{c^2}\right)^{\frac{1}{2}}\left[\left(1+\dfrac{q^2}{c^2}\right)^{\frac{1}{2}}-1\right]+\dfrac{WD}{c^2[0-(W^2/c^2)]^{\frac{1}{2}}}}{\left(1+\dfrac{q^2}{c^2}\right)^{\frac{1}{2}}}. \quad (10)$$

If we imagine this process carried through for a series of clocks fixed on the platform we end up by having an array of clocks all running with the frequency $\nu_0[1-(W^2/c^2)]^{\frac{1}{2}}$, but each *set back* by an amount which is a function of its distance, the velocity of the platform through the medium, and the rod-clock-quotient of the moved clock.

We can simplify the expression for Δ, by inserting the value of τ from the relation $\tau=D/Y=D'/Y[1-(W^2/c^2)]^{\frac{1}{2}}$, taking the value of Y from (8) which gives

$$\tau=\left[D'\left[\left(1+\frac{q^2}{c^2}\right)^{\frac{1}{2}}+\frac{Wq}{c^2}\right]\right]\Big/\left[q\left(1-\frac{W^2}{c^2}\right)^{\frac{1}{2}}\right], \quad (11)$$

and this in (10) gives

$$\Delta=\frac{D'}{q}\left[\left(\left(1+\frac{q^2}{c^2}\right)^{\frac{1}{2}}-1\right)+\frac{Wq}{c^2}\right]. \quad (12)$$

We now seek the expression for the velocity, V (Fig. 1), of any body moving uniformly with respect to the platform, as measured by two fixed clocks at a and b, set by the above procedure. Since $V=D/t$ we can obtain this by finding D and t. For D we have $D=D'[1-(W^2/c^2)]^{\frac{1}{2}}$. We get t from the time indicated by the clock at b when the moving body passes it, having passed a at the indicated time zero. This is given by the relation

$$t'=t[1-(W^2/c^2)]^{\frac{1}{2}}-\Delta$$

or

$$t=(t'+\Delta)\Big/\left(1-\frac{W^2}{c^2}\right)^{\frac{1}{2}}. \quad (13)$$

[8] This is one of the commonest ways of measuring velocity. It is used by the mariner with his chronometer, by the automobilist in traversing the "measured mile," and by the train traveller who counts telegraph poles passed in the interval given by the watch in his hand. By it the measured velocity of light is infinity.

Substituting the value of Δ from (12) and putting Q_2 for D'/t' we get

$$V = \frac{Q_2\left(1-\frac{W^2}{c^2}\right)}{1+\frac{Q_2 W}{c^2}+\frac{Q_2}{q}\left[\left(1+\frac{q^2}{c^2}\right)^{\frac{1}{2}}-1\right]}. \qquad (14)$$

We now find the expression for the velocity of light. For the case considered the velocity of light with respect to the platform is $c-W$. Putting this in place of V we get, for the rod-clock-quotient

$$Q_2 = (c) \Big/ \left[1-\frac{c}{q}\left(\left(1+\frac{q^2}{c^2}\right)^{\frac{1}{2}}-1\right)\right], \qquad (15)$$

a constant, independent of W.

This constant, if we had performed the measurement in ignorance of the F.L.L. contractions, *would have been interpreted as the velocity of light.* It is not equal to c.

We get the velocity of light by solving (15) for c. This gives us

$$c = [2Q_2(Q_2-q)] \Big/ [2Q_2-q]. \qquad (16)$$

This formula is the goal of the present paper. It gives the velocity of light as measured by signals in one direction, in terms of the observed quantities, namely two rod-clock-quotients. These are the rod-clock-quotient Q_2, involving the difference of readings of the two fixed clocks as the light signal passes them, and the rod-clock-quotient q of the moved clock used for setting the fixed clocks.

Again, as with the out-and-back signals, we do not get the velocity of light with respect to the platform ($c\pm W$), but the velocity of light with respect to the medium. Note that if the moved clock is moved so slowly that $q\cong 0$ we approximate to

$$c = Q_2.$$

THE LORENTZ TRANSFORMATIONS

From the preceding section we see that the rod-clock-quotient Q_2, for a unidirectional light signal, has the interesting property of being the same (if a definite q is adhered to) no matter what the velocity of the body on which it is measured. Consequently if we initiate a light signal at a given point in the light transmitting medium at which the initial points of a series of moving platforms are at that instant in coincidence, the spherical wave expanding in the medium will be ascribed the same Q_2 on each platform. That is, the equations describing the wave, as set up for each platform, will have *the same form in terms of* Q_2, as a spherical wave in the stationary medium *in terms of* c. We can thus set up a family of equations, applying to a series of relatively moving bodies, in which Q_2 is expressed in rod and clock measurements peculiar to each body, and thus derive a relationship between these measurements, involving the relative measured motions of the bodies, and Q_2.

This is the procedure used in expositions of the Special Theory of relativity to derive the Lorentz transformations,[6] which are, according to our development, properly expressed in terms of Q_2. By the further step of agreeing to move the clocks used for setting "local times" at infinitesimal rates ($q\cong 0$), Q_2 approximates to c, and the usual form of the Lorentz transformations is obtained. Incidentally, by adopting this convention, we permit, as a practical substitute for the slowly moved clock for setting purposes, the use of light signals ascribed the (false) velocity c.

THE SPECIAL THEORY OF RELATIVITY

The Special Theory of Relativity, the formulae of which are developed from consideration of light signals sent in one direction, is an example of the importance of the precise definition of the measuring processes involved, and of the distinction here drawn between "velocity" and the "rod-clock-quotients." The paradox confronting our elementary and traditional concept of velocity by the proposition that the velocity of light is constant, both with respect to the medium and to a body moving through the medium, is avoided by the analysis and nomenclature here employed.

The "constancy of the velocity of light" which figures in current text-book treatments of the theory, should, according to the present analysis, be more accurately described as *two* "constancies" namely of the two rod-clock-quotients Q_1 and Q_2.

[6] W. H. McCrea, *Relativity Physics* (Methuen, London, and Company, Ltd. 1935), p. 8.

The rod-clock-quotient Q_1, which is obtained in the out-and-back measurement of light signals, using a single clock, has the constant value c, which is the velocity of light measured by rods and clocks stationary in the medium, or by formulae (6) and (16). The rod-clock-quotient Q_2, which is obtained in the one-way measurement of light signals, using two clocks, is a constant for any one choice of the rate of motion (q) of the auxiliary clock used for setting the two fixed clocks, but varies with q, and is not in general equal to c.

The single value for the "velocity of light" which figures in developing the formulae of the theory is due to the specification that the rod-clock-quotient for one-way light signals, Q_2, shall be the same as that for out-and-back signals, Q_1.[7] This specification (sometimes called "the adjustment of constants") turns out to mean that the only measurement of one-way light signals which is to be allowed is one where the clock used for setting is moved at an infinitesimal rate. This reservation on permissible measurements vitiates the claim that the "velocity of light" is an "absolute" factor in the Special Theory of Relativity.

LIGHT SIGNALS IN ONE DIRECTION THROUGH A REFRACTING MEDIUM

In the foregoing we have been concerned solely with light signals in the ether. The case where the light is transmitted through a refracting medium can be handled in the same manner. It is only necessary to insert for V in Eq. (14) the velocity of light in the refracting material, in place of $c-W$, the velocity in the ether, as was done in deriving Eq. (15).

[7] In Einstein's original presentation he declared the value "c" for the velocity of light to be "firmly established by experiment." This was not a legitimate citation, for he was discussing the *one-way* transmission of signals, while the only existing experimental values were those from *out-and-back* measurement.

For the velocity of light in a refracting material moving through the ether, two formulae are available, derived from the theory of electrons, one due to Lorentz,[8] the other to Larmor.[9] The formula of Lorentz, aimed at explaining the observed Fresnel drag coefficient, neglects all terms in the second order of v/c, and while adequate for its purpose, is not suited for the present discussion, in which the second order terms are an essential factor. The formula of Larmor, incorporating the F.L.L. contractions and correct to the second order, is expressed in the exact form here needed.

Larmor's formula for the velocity[10] of light in a refracting material, moving with velocity W with respect to the ether, in terms of coordinates fixed in the refracting material, is

$$V=\frac{c}{\mu}\left(1-\frac{W^2}{c^2}\right)\Big/\left(1+\frac{W}{c\mu}\right) \qquad (17)$$

where c/μ is the velocity of light in the material when stationary in the ether.[11]

Putting this for V in (14) we get

$$Q_2=\frac{c}{\mu}\Big/\left[1-\frac{c}{\mu q}\left[\left(1+\frac{q^2}{c^2}\right)^{\frac{1}{2}}-1\right]\right] \qquad (18)$$

a constant independent of W.

[8] P. Drude, *Lehrbuch der Optik* (Hirzel, Leipzig, 1900) p. 426.

[9] J. Larmor, "Aether and Matter," (Cambridge University Press, 1900) p. 178.

[10] Larmor uses the term "velocity" in exactly the sense adhered to in the present paper.

[11] If we transform our origin of coordinates to the light transmitting medium instead of the refracting material we get the relation

$$V+W=\left[\frac{c}{\mu}\left(1-\frac{W^2}{c^2}\right)\Big/\left(1+\frac{W}{c\mu}\right)\right]+W=\left(\frac{c}{\mu}+W\right)\Big/\left(1+\frac{W}{c\mu}\right).$$

This formula is commonly credited to the Special Theory of Relativity. That the considerably earlier formula of Larmor (17), derived from analysis of the effect of moving charges on the velocity of electromagnetic waves through the surrounding ether, is identical in content, appears to have been generally overlooked.