# Historical Note on the Rate of a Moving Atomic Clock

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(Received May 7, 1947)

The history of the idea of variation of frequency with velocity is followed through Voigt, Larmor, Lorentz, and Einstein. The Michelson-Morley experiment is explainable by any contraction of dimensions in the ratio $(1-v^2/c^2)^{\frac{1}{2}}:1$ along and transverse to the direction of motion. To each contraction corresponds a different value of frequency change. The theoretical speculations pointing to the relation $\nu_m=\nu_0(1-v^2/c^2)^{\frac{1}{2}}$ are discussed, together with the significance of the experimental test by means of canal rays.

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THE first suggestion that a "natural" clock would alter its rate on motion appears in a paper by Voigt[1] in 1887. Put in modern terminology his statement translates "We have therefore a glowing plane progressing with velocity $c$, vibrating with the period $T'=T/(1-v^2/c^2)$." The coefficient of $T$, it will be observed, is not the same as that proposed by Larmor, Lorentz, and Einstein, and found by experiment,[2] namely $1/(1-v^2/c^2)^{\frac{1}{2}}$. The history and significance of this coefficient and of its exact value, are of some interest.

H. A. Lorentz, following his and Fitzgerald's *ad hoc* postulation of a contraction of material bodies in their direction of motion, to account for the null result of the Michelson-Morley experiment, undertook an extension of the idea "to reduce the equations for a moving system to the ordinary formulae that hold for a system at rest." To do this he introduced new variables:

$$\begin{aligned} x'&=l(x-vt)/(1-v^2/c^2)^{\frac{1}{2}},\\ y'&=ly,\\ z'&=lz,\\ t'&=l(t-vx/c^2)/(1-v^2/c^2)^{\frac{1}{2}}, \end{aligned} \tag{1}$$

where $l$ is a numerical coefficient, a function of the velocity of translation, whose value is unity for $v=0$. The decision on the value of $l$ to insert was the subject of considerable study by Lorentz, of which more below.

In a note to his *Theory of Electrons* (1913) Lorentz refers as follows[3] to Voigt's publication: "In a paper which to my regret has escaped my notice all these years, Voigt has applied a transformation equivalent to the formulae (287) and (288). The idea of the transformations used above might therefore have been borrowed from Voigt and the proof that it does not alter the form of the equations for the free ether is contained in his paper."

On examining Voigt's original paper it will be found that he did not express his findings in the general form given in (1) but selected for discussion the equations

$$\begin{aligned} x'&=x-vt,\\ y'&=y(1-v^2/c^2)^{\frac{1}{2}},\\ z'&=z(1-v^2/c^2)^{\frac{1}{2}},\\ t'&=t-vx/c^2, \end{aligned} \tag{2}$$

that is, he chose the value $(1-v^2/c^2)^{\frac{1}{2}}$ for Lorentz' $l$. The reason for this choice is not clear.

It has been frequently asserted that the value of $l$ is of no importance. Thus Cunningham[4] states: "The effect of the factor $l$ indicates only a uniform magnification of the scales of space and time, or what is the same thing, a change of units. It does not introduce any essential modification;" and Silberstein:[5] "The value of such a coefficient is essentially, from the physical standpoint, a matter of indifference." While this is so, if the only purpose of the transformations is to explain the Michelson-Morley experiment, *it is not the case if we are to predict the rate of a moving clock.*

This can be made clear by a graphical presentation. Consider the two end mirrors of a

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[1] Voigt, "Über das Doppler'sche Princip," Göttinger Nachrichten (March 10, 1887).

[2] Ives and Stillwell, "An experimental study of the rate of a moving atomic clock," J. Opt. Soc. Am. 28, 215 (1938), and **31**, 369 (1941).

[3] Lorentz, *The Theory of Electrons* (Columbia University Press, New York, 1906), reprint by Stechert, 1923, p. 198.

[4] Cunningham, *The Principle of Relativity* (Cambridge University Press, Teddington, England, 1914), p. 50.

[5] Silberstein, *The Theory of Relativity* (Macmillan Company, London, 1914), p. 119.

Michelson interferometer as lying at the ends of two perpendicular radii of a sphere (Fig. 1) moving in the direction shown, with the velocity $v$ with respect to the medium transmitting the light waves with velocity $c$. Then, if the radius $=a$, the length of the "vertical" light path, out and back is $2a/(1-v^2/c^2)^{\frac{1}{2}}$, of the "horizontal" path is $2a/(1-v^2/c^2)$, and the times of signal transit are $1/c$ times these. If the sphere is replaced by an ellipsoid with radius in the direction of motion in the ratio $(1-v^2/c^2)^{\frac{1}{2}}$ to the radii at right angles, the signal transit times will be identical, and the Michelson-Morley result is accounted for.

This ratio may be secured by an infinite series of actual dimensions corresponding to different values of $l$ in Eq. (1). Let us investigate several of these.

First take Voigt's choice of $l$. This corresponds to an oblate spheroid of radius $a$ in the direction of motion, and of radius $a/(1-v^2/c^2)^{\frac{1}{2}}$ at right angles thereto (Fig. 2a). For this the time of signal transit (in both directions) is $2a/c(1-v^2/c^2)$. Using the successive returns of the reflected signal to the origin as clock "ticks" the clock period is

$$T_m=T_s/(1-v^2/c^2)$$

as found by Voigt.

Next, take the value of $l$ finally chosen by Lorentz, namely, $l=1$. This is another oblate spheroid of radius $a(1-v^2/c^2)^{\frac{1}{2}}$ in the direction of motion, of radius $a$ at right angles thereto (Fig. 2b). The clock period is

$$T_m=T_s/(1-v^2/c^2)^{\frac{1}{2}}.$$

As another case take the spheroid of the same volume as the stationary sphere (Fig. 2c), a

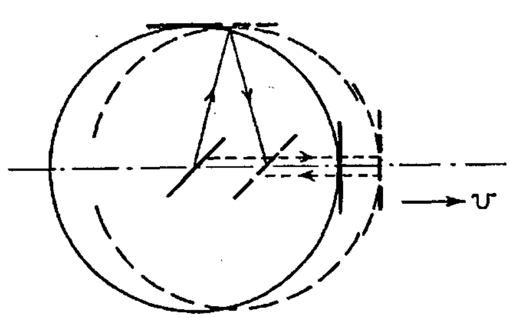


FIG. 1.

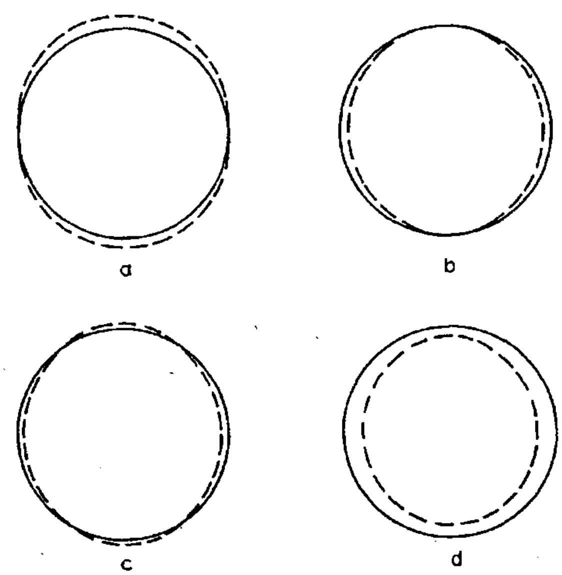


FIG. 2.

form which Lorentz considered attractive for the deformable electron. Here the radius in the direction of motion is $a(1-v^2/c^2)^{\frac{1}{3}}$, at right angles thereto $a/(1-v^2/c^2)^{1/6}$. The value of $l$ is $(1-v^2/c^2)^{1/6}$, and the clock period is

$$T_m=T_s/(1-v^2/c^2)^{\frac{2}{3}}.$$

As a final case consider the spheroid contracted in the direction of motion by the factor $(1-v^2/c^2)$, and by the factor $(1-v^2/c^2)^{\frac{1}{2}}$ at right angles thereto (Fig. 2d). For this the value of $l$ is $(1-v^2/c^2)^{-\frac{1}{2}}$ and the clock period is $T_m=T_s$.

With such a contraction the Michelson-Morley experiment is accounted for, *but there is no change in clock rate.*

For comparison these several theoretical effects are plotted (Fig. 3) in terms of the shift of center of gravity of Doppler lines for approaching and receding canal rays, emitting radiation $\lambda$4861, along with the results obtained experimentally.[6] These latter decide unequivocally for $l=1$.

While the matter of the exact value of $l$ has been frequently dismissed as unimportant, it was not so considered by Lorentz, who discussed it at some length. No answer is obtainable

[6] These experimental results are assembled from the two papers given in reference 2, which may be consulted for the units and symbols employed.

through the Michelson-Morley experiment alone.[7] Ultimately Lorentz picked the value $l=1$ by consideration of the variation of mass with velocity as derived from electromagnetic speculations. He was then able to state that his complete system of equations, with $l=1$, "enable us to predict that no experiment made with a terrestrial source of light will ever show us the influence of the earth's motion." Einstein, starting with this conclusion (which was practically Lorentz' working hypothesis from the start), and elevating it to a new principle of physics, was able, by working backward, to deduce the contraction factor $(1-v^2/c^2)^{\frac{1}{2}}$.

The attitude of Einstein toward the consequences of his line of thought was very different in respect to the question of clock rate from that of his predecessors. Voigt gives no reason for developing his transformations. He does not mention the Michelson-Morley experiment. His paper appears to be a mathematical exercise. The conclusion that a moving clock *would* actually behave as is called for by his above quoted statement is not ventured. Lorentz used "local time" "for the sake of facilitating our mode of expression," but never in his original presentations gave to clock behavior the objective reality he maintained for the Fitzgerald-Lorentz contraction. Larmor, who also used local time, came closer when he stated:[8] "The change of time variable in the comparison of radiations in the fixed and moving systems involves the Doppler effect on the wave-length," but left the matter there without further elucidation. Einstein was bold enough to believe what the equations said, and to back up his belief by the prediction that a second-order Doppler effect in canal rays already being sought for by Stark, on rather general grounds,[9] would in fact reveal the relation of frequencies $\nu_m=\nu_s(1-v^2/c^2)^{\frac{1}{2}}$. He entitled his note "Über die Möglickeit einer neuen Prüfung des Relativitätsprinizips,"[10] and accordingly for some thirty years thereafter this observation on canal rays, if and when the experimental difficulties in the way of its achievement could be overcome, figures in discussions of the subject as an *experimentum crucis*.

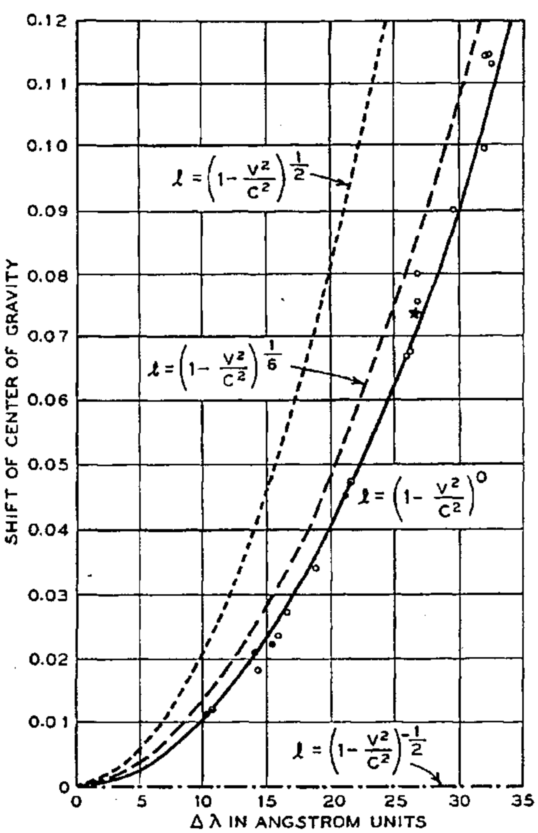


FIG. 3.

The existence of the motional variation of clock rate, *and its exact value*, are interpretable, from the above account, as decisive verification of the theoretical work of Voigt, Larmor, Lorentz, and Einstein. The whole matter can however, be considered in relation to a new theoretical approach[11] owing nothing to the Michelson-Morley experiment, which latter is the undoubted spark which activated the above investigators.

In this latest approach, radiation is required to conform to Maxwell's equations referred to a wave-transmitting medium, and the requirement

[7] Kennedy and Thorndyke, in their experiment with a Michelson-Morley interferometer with arms of unequal length ["Experimental proof of the relativity of time," Phys. Rev. 42, 400 (1932)] *assume* the linear contraction $(1-v^2/c^2)^{\frac{1}{2}}$ in the direction of motion. Had they assumed the contractions of Fig. 2d they would have "proved" the *non*-relativity of time.

[8] Larmor, *Aether and Matter* (Cambridge University Press, Teddington, England, 1900), p. 177.

[9] Stark, "Über die Lichtemission der Kanalstrahlen in Wasserstoff," Ann. d. Physik 13, 401 (1906); in particular part III on the probable variation of wave-length according to a function of $v^2/c^2$.

[10] Einstein, Ann. d. Physik 12, 197 (1907).

[11] Ives, "Derivation of the Lorentz transformations," Phil. Mag. (7) 36, 392 (1945).

is imposed that the laws of conservation of energy and momentum shall hold universally and exactly in reactions between radiation and matter. The problem is attacked by study of the mechanical interactions of radiation and matter encountered in the radiation pressure of Maxwell. It is found that the above requirements first demand the variation of mass with velocity, and in turn the variations of dimensions and clock rate according to the factor $(1-v^2/c^2)^{\frac{1}{2}}$ appropriately placed. The impossibility of detecting the earth's motion through the radiation-transmitting medium follows, as in Lorentz' theory, as a consequence. The nul result of the Michelson-Morley experiment appears not as a postulate (as in the special theory of relativity) but as a corollary of older physical laws of wide generality, demanding no "new principle."[12] In the same list of corollaries are the variation of mass and clock rate.

Of experimental tests for establishing the reality of these contractions, however arrived at, the most crucial is the variation of clock rate; for while the Michelson-Morley experiment has the uncertainty that it might be explainable by an entrained ether (Stokes), and the electrical experiments on variation of mass might be explained by variation of charge with velocity, as is recurrently proposed, no comparable alternatives appear to weaken the conclusiveness of the experiment on clock rate.

[12] Newton, *Rules of Reasoning in Philosophy*, "We are to admit no more causes of natural things than such as are both true and sufficient to explain their appearance," Rule 1.

JOURNAL OF THE OPTICAL SOCIETY OF AMERICA VOLUME 37, NUMBER 10 OCTOBER, 1947