The Doppler Effect from Moving Mirrors

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PREVIOUS to the discovery of the Doppler effect in canal rays the only laboratory demonstration of the Doppler effect with light was that of Bélopolski and Galitzen[1] in which light was reflected back to the source from a rapidly moving mirror. In the experiment as performed, the light was reflected not once but several times, whereby the effect was made large enough for satisfactory measurement with an echelon spectroscope.

This Doppler effect, sometimes referred to as the "artificial Doppler effect" or "a sort of Doppler effect," differs from that observed with canal rays, or in the case of the stars, in that it cannot be expressed in terms of the relative motion of light source and observer, since there is no relative motion. It must be treated in terms of the motion of the radiation in the space between the light source, the mirrors, and the observer.

The formula for the frequency of the light returned to the source by a mirror moving away from it and oriented normally to the direction of motion, as obtained directly from wave theory, and given in several treatises on optics,[2] is

$$\nu' = \nu\left[\frac{c-v}{c+v}\right], \tag{1}$$

where ν is the frequency of the source, c the velocity of light, and v the velocity of the mirror. If the light is reflected back and forth a number of times, as by placing a stationary mirror at the source, the formula becomes

$$\nu' = \nu\left[\frac{c-v}{c+v}\right]^n, \tag{2}$$

where n is the number of incidences of light on the moving mirror.

[1] Bélopolski and Galitzen, Acad. Sci. St. Petersburg Bull. 8, 213 (1907).

[2] Alfred O'Rahilly, *Electromagnetics* (Longmans, Green and Co., 1938), p. 339.

This formula is developed for the case of source and observer being stationary in the ether. It is of interest to investigate the phenomena when the whole system is in uniform motion, as when the light source and observer are on a platform moving with the velocity V in the direction normal to the moving mirror. The formula for this case is simply obtained by double application of the formula (1). We note that a mirror placed at the moving light source would reflect the radiation incident on it with a frequency changed in the ratio $(c+V/c-V)$. This frequency is obviously that at which the radiation would strike the mirror. Applying this factor to the radiation returning from the moving mirror leads to the formula

$$\nu' = \nu\left[\frac{c+V}{c-V}\right]^n \left[\frac{c-(v+V)}{c+(v+V)}\right]^n, \tag{3}$$

where $(v+V)$ is the absolute velocity of the moving mirror.

An interesting question, which it is the purpose of this note to discuss, is whether this experiment would be fitted, as casual inspection of formula (3) suggests, to detect absolute motion through the ether. Or, looking at the question from another angle, what factors are available which would make this experiment, like all other optical experiments, yield a "null" result? It is not immediately obvious how the Fitzgerald contraction, and the Larmor-Lorentz change of clock rate, which are invoked for the Michelson-Morley experiment, can be operative here. The Fitzgerald contraction is not directly applicable because no lengths appear in the formula. The change of frequency of the light source in motion will cause no change in the observed effect since this frequency is itself our reference standard.

Inspection of formula (3) shows that there is only one factor which is subject to experimental manipulation, namely v, the velocity of the moving mirror. This velocity must be set *by measurement;* and in variations in its absolute value when measured on platforms moving at different velocities must be found the desired modification of the formula. Calling this velocity v' when measured on the moving platform, and retaining v for the value on a stationary platform we seek the value of v' which shall make

$$\left(\frac{c+V}{c-V}\right)\left(\frac{c-v'-V}{c+v'+V}\right)=\frac{c-v}{c+v}. \tag{4}$$

Solving this relation we find

$$v'=\frac{v\left[1-\dfrac{V^2}{c^2}\right]}{1+\dfrac{Vv}{c^2}}. \tag{5}$$

We next investigate by what process of measurement we would be led to move the mirror at the velocity v on a stationary platform, and at the velocity

$$\frac{v\left[1-\dfrac{V^2}{c^2}\right]}{1+\dfrac{Vv}{c^2}}$$

on a platform moving with the velocity V. In order to do this, we consider the elementary method of velocity measurement consisting in counting divisions passing by a clock. The only way this process can lead to different results under different conditions, such as different velocities of the system, is for the length of the divisions, or the interval between clock ticks, or both, to vary. Let the divisions be on the mounting of the mirror, and let the clock be fixed at a point on the platform. Now with the platform stationary the velocity v will be read as

$$\frac{v}{L(v)}, \tag{6}$$

where $L(v)$ is some function of v representing the change in length of the divisions on the mirror mounting caused by its motion. With the platform moving we have for the corresponding measurement of velocity

$$\frac{\dfrac{v\left[1-\dfrac{V^2}{c^2}\right]}{1+\dfrac{Vv}{c^2}}}{F(V)\cdot L\left[\dfrac{v+V}{1+\dfrac{vV}{c^2}}\right]}, \tag{7}$$

where $F(V)$ is the factor by which the frequency of the clock on the moving platform is altered, and

$$L\left[\frac{v+V}{1+\dfrac{vV}{c^2}}\right]$$

is the same length altering factor as before, noting that the absolute velocity of the mirror is

$$v'+V=\frac{v\left[1-\dfrac{V^2}{c^2}\right]}{1+\dfrac{Vv}{c^2}}+V=\frac{v+V}{1+\dfrac{vV}{c^2}}. \tag{8}$$

We then put down as the result of setting the mirror to move on the moving platform at the same *measured* velocity as on the stationary platform

$$\frac{v}{L(v)}=\frac{v\left[1-\dfrac{V^2}{c^2}\right]}{\left[1+\dfrac{vV}{c^2}\right]\cdot F(V)\cdot L\left[\dfrac{v+V}{1+\dfrac{vV}{c^2}}\right]}. \tag{9}$$

Making use of the identity

$$1+\frac{vV}{c^2}=\frac{\left[1-\dfrac{v^2}{c^2}\right]^{\frac{1}{2}}\left[1-\dfrac{V^2}{c^2}\right]^{\frac{1}{2}}}{\left[1-\dfrac{(v+V)^2}{c^2\left[1+\dfrac{vV}{c^2}\right]^2}\right]^{\frac{1}{2}}} \tag{10}$$

(9) becomes

$$\frac{\left[1-\frac{v^2}{c^2}\right]^{\frac{1}{2}}}{L(v)}=\frac{\left[1-\frac{V^2}{c^2}\right]^{\frac{1}{2}}}{F(V)}\cdot\frac{\left[1-\frac{(v+V)^2}{c^2\left[1+\frac{vV}{c^2}\right]^2}\right]^{\frac{1}{2}}}{L\left[\frac{v+V}{1+\frac{Vv}{c^2}}\right]}, \quad (11)$$

from which

$$L(v)=\left[1-\frac{v^2}{c^2}\right]^{\frac{1}{2}},$$

$$L\left[\frac{v+V}{1+\frac{vV}{c^2}}\right]=\left[1-\frac{(v+V)^2}{c^2\left[1+\frac{Vv}{c^2}\right]^2}\right]^{\frac{1}{2}}, \quad (12)$$

$$F(V)=\left[1-\frac{V^2}{c^2}\right]^{\frac{1}{2}}.$$

These relations state that (for the experiment to give a "null" result), lengths and clock frequencies must be contracted in the ratio

$$\left[1-\left[\frac{\text{velocity}}{\text{velocity of light}}\right]^2\right]^{\frac{1}{2}}.$$

Since these are the alterations of length and clock rate to which is ascribed the result of the Michelson-Morley experiment, it follows that the Doppler effect with moving mirrors, if the experiment could be performed with adequate sensitivity, would likewise give a null result insofar as detecting absolute motion is concerned. The manner in which the contractions enter is however quite different. From these contractions it is possible to derive the Lorentz transformations,[3] and this elementary derivation is an example of Bateman's remark[4] that these transformations are derivable from study of the reflection of light from a moving mirror.

[3] H. E. Ives, J. Opt. Soc. Am. **27**, 263 (1937).

[4] H. Bateman, Bull. Nat. Research Council, Dec., 1922, p. 110.